Advanced Studies in Contemporary Mathematics
30 (2020), No. 1, pp. 1 – 4

www.jangjeon.or.kr
http://dx.doi.org/10.17777/ascm2020.30.1.1

RESTRICTION OF A GUICHARDET–WIGNER PSEUDOCHARACTER ON A SIMPLY CONNECTED SIMPLE HERMITIAN SYMMETRIC LIE GROUP TO A SIMPLY CONNECTED SIMPLE HERMITIAN SYMMETRIC SUBGROUP

A. I. Shtern

Abstract. A simple example shows that the restriction of a Guichardet–Wigner pseudocharacter on a simply connected simple Hermitian symmetric Lie group to a simply connected simple Hermitian symmetric subgroup can be a Guichardet–Wigner pseudocharacter on the subgroup. This poses the natural problem of whether or not such a restriction can give a zero real character.

For the definitions and details concerning pseudocharacters and quasicharacters, see [1–3].

§ 1. Introduction

The role of Guichardet–Wigner pseudocharacters on Hermitian symmetric connected Lie groups in the description of locally bounded finite-dimensional pseudorepresentations of these groups is significant (see [1–4]). In this connection, there is a natural problem concerning the restriction of a Guichardet–Wigner pseudocharacter on a simply connected simple Hermitian symmetric Lie group to a simply connected simple Hermitian symmetric Lie subgroup. In principle, this restriction can be either a Guichardet–Wigner pseudocharacter on the subgroup or a zero real character. The present paper shows that, for one of the simplest situations, the first possibility holds.

2010 *Mathematics Subject Classification.* Primary 22A99, Secondary 22A25, 22A10.
Key words and phrases. Guichardet–Wigner pseudocharacter, Hermitian symmetric semisimple Lie group.

§ 2. Preliminaries

Recall the definition of the main object of the paper.

Definition. Let G be a connected simply connected simple Lie group whose center is infinite (and thus, as is well known, the corresponding symmetric space is Hermitian symmetric; see, e.g., [1]) and let K be an analytic subgroup of G corresponding to a maximal compact Lie subalgebra of the Lie algebra of the Lie group G. Let us isomorphically identify the center Z_K of the analytic group K with the additive group of the real field $\mathbb{R}$ (for instance, by the arc length on the one-parameter subgroup defined by the subgroup Z_K). Consider the Iwasawa decomposition $G = KAN$ related to the group K. Let A be the Abelian group and N the nilpotent group in this decomposition and let

$$g = k(g)an, \qquad g \in G, \quad k(g) \in K, \quad a \in A, \quad n \in N,$$

be the corresponding decomposition of an element $g \in G$. As is well known, the mapping

$$\varpi\colon g \mapsto k(g), \qquad g \in G,$$

taking every element $g \in G$ to the "compact" component $k(g) \in K$ of the Iwasawa decomposition is continuous. Consider the composition ψ of the mapping

$$\varpi\colon g \mapsto k(g), \qquad g \in G,$$

and the continuous projection π taking every element $k \in K$ to its central component $z(k) \in Z_K$. As was proved in [1–3], this composition

$$\psi = \pi \circ \varpi$$

defines a quasicharacter on G. The pseudocharacter θ corresponding to this quasicharacter is referred to as the *Guichardet–Wigner pseudocharacter*, cf. [1].

§ 3. Main theorem

Theorem. *Let G be the universal covering group of* $\mathrm{SU}(m,n)$*, $m,n \in \mathbb{N}$, $m \geq 2$, let $H \subset G$ be the group isomorphic to the universal covering group of* $\mathrm{SU}(m-1,n)$*, where the matrices of the natural homomorphism of H into* $\mathrm{SU}(m,n)$ *are distinguished by the condition that the entry of every matrix* *in the image of this mapping at the upper left corner is equal to one. Then the restriction of every Guichardet–Wigner pseudocharacter on G to H is a nontrivial Guichardet–Wigner pseudocharacter on H.*

Proof. Let us use the explicit formulas of [5], p. 286, for the corresponding 2-cocycle; these formulas claim that, for the function $v'(g) = \det g_{11}$, where $g \in G$ and $g = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}$ according to $p+q$ block decomposition, we have the formula

$$f\colon (g_1, g_2) \mapsto \frac{1}{2\pi} \arg(v'(g_1)v'(g_2)v'(g_1g_2)^{-1}), \quad g_1, g_2 \in G,$$

and there is a quasicharacter (see [1]) corresponding to a continuous branch of the right-hand side which exists by the Guichardet–Wigner theorem; the pseudocharacter corresponding to this quasicharacter is a Guichardet–Wigner pseudocharacter on G (defined up to a constant multiple). It follows immediately from the above formula that the restriction of f to H is unbounded (this can be seen from the consideration of a subgroup $\mathrm{SU}(1,1)$ defined by the last p-index and the first q-index), and hence the pseudocharacter corresponding to $f|_H$ is nonzero. Since every pseudocharacter on a simple Hermitian symmetric simply connected Lie group is a nonzero multiple of a Guichardet–Wigner pseudocharacter, it follows that the restriction of the Guichardet–Wigner pseudocharacter on G to the subgroup H is a Guichardet–Wigner pseudocharacter on H.

§ 4. Concluding remarks

Problem. *There is a natural problem concerning the restriction of a Guichardet–Wigner pseudocharacter on a simply connected simple Hermitian symmetric Lie group to a simply connected simple Hermitian symmetric Lie subgroup. The above theorem shows that this restriction can be a Guichardet–Wigner pseudocharacter on the subgroup. The problem is, whether or not this restriction can be a zero real character on the subgroup.*

Acknowledgments

I thank Professor Taekyun Kim for the invitation to publish this paper in the Advanced Studies of Contemporary Mathematics.

Funding

The research was partially supported by the Scientific Research Institute of System Analysis, Russian Academy of Sciences (FGU FNTs NIISI RAN), theme NIR 0065-2019-0007.

References

1. A. I. Shtern, *Finite-dimensional quasi-representations of connected Lie groups and Mishchenko's conjecture*, J. Math. Sci. **159** (2009), no. 5, 653–751.
2. A. I. Shtern, *Remarks on finite-dimensional locally bounded finally precontinuous quasirepresentations of locally compact groups*, Adv. Stud. Contemp. Math. (Kyungshang) **20** (2010), no. 4, 469–480.
3. A. I. Shtern, *A version of van der Waerden's theorem and a proof of Mishchenko's conjecture on homomorphisms of locally compact groups*, Izv. Math. **72** (2008), no. 1, 169–205.
4. A. I. Shtern, *Locally bounded finally precontinuous finite-dimensional quasirepresentations of connected locally compact groups*, Mat. Sb. **208** (2017), no. 10, 149–170; English transl., Sb. Math. **208** (2017), no. 10, 1557–1576.
5. A. Guichardet and D. Wigner, *Sur la cohomologie réelle des groupes de Lie simples réels*, Ann. Sci. École Norm. Sup. (4) **11** (1978), no. 2, 277–292.

Department of Mechanics and Mathematics,
Moscow State University,
Moscow, 119991 Russia, and
Scientific Research Institute of System Analysis, Russian Academy of Sciences (FGU FNTs NIISI RAN),
Moscow, 117312 Russia
E-mail: aishtern@mtu-net.ru, rroww@mail.ru